Final Project – ME 314, Machine Dynamics

Model of a Trebuchet

Description

My original proposal was of a different project but adhering to the feedback, I decided to go ahead with the trebuchet. A trebuchet has three degrees of freedom, one of which is initially constrained as the projected mass, M_2 rests on the floor. As the counterweight descends, the contact is lost and the M_2 is launched when the two links align. The hinged arm has a centre of mass at an offset of L_m. M_2 is a rigid body, circular in shape with three configuration parameters.

After the projectile is launched, M_2 impacts a horizontal line at a height of 2.5m from the base. To show realistic results, a coefficient of restitution with a value of 0.5 is added to the impact update law. The angular velocity of M_2 just changes direction owing to friction. This friction force calculation is not included in this project.

Transformations and drawing

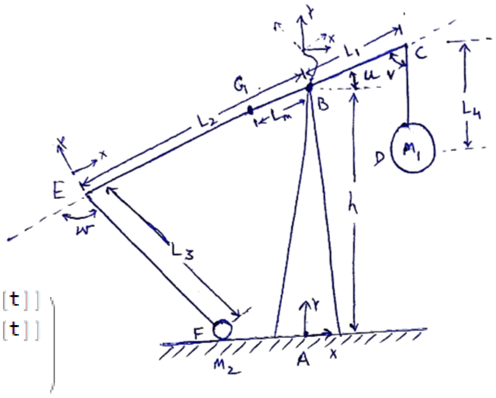

Figure 1 Diagram of the system

$T_{AG} = T_{AB}.T_{BB'}.T_{B'G}$

$$T_{AG} = \begin{pmatrix} \text{Cos}[u[t]] & -\text{Sin}[u[t]] & 0 & -0.5\,\text{Cos}[u[t]] \\ \text{Sin}[u[t]] & \text{Cos}[u[t]] & 0 & -0.5\,\text{Sin}[u[t]] \\ 0 & 0 & 1 & 0. \\ 0 & 0 & 0 & 1. \end{pmatrix}$$

$T_{AD} = T_{AB}.T_{BB'}.T_{B'C}.T_{CC'}.T_{C'D}$

$$T_{AD} = \begin{pmatrix} \text{Cos}[u[t]+v[t]] & -\text{Sin}[u[t]+v[t]] & 0 & \text{Cos}[u[t]]-\text{Cos}[u[t]+v[t]] \\ \text{Sin}[u[t]+v[t]] & \text{Cos}[u[t]+v[t]] & 0 & \text{Sin}[u[t]]-\text{Sin}[u[t]+v[t]] \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$T_{AF} = T_{AB}.T_{BB'}.T_{B'E}.T_{EE'}.T_{E'F}$

$$T_{AF} = \begin{pmatrix} \text{Sin}[u[t]+w[t]] & \text{Cos}[u[t]+w[t]] & 0 & -3\,\text{Cos}[u[t]]-4\,\text{Cos}[u[t]+w[t]] \\ -\text{Cos}[u[t]+w[t]] & \text{Sin}[u[t]+w[t]] & 0 & -3\,\text{Sin}[u[t]]-4\,\text{Sin}[u[t]+w[t]] \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Formulation

Using the rigid body transformations above, I derived the velocities using $G^{-1}\dot{G}$ and then calculating the kinetic energy of the system. The potential energy is calculated using the heights from the transformations. For the initial part of the simulation, M_2 slides on the ground. This is modelled as a constraint. For the second part where the mass is swinging, the system is unconstrained. The mass is released when the two arms align. The mass in mid-air is given by [q[t], x[t], y[t]]. The impact occurs when the periphery of M_2 satisfies the constraint. This is calculated as $T_{AM}.T_{MP}$, where T_{MP} represents the transformation from the centre of the body to the periphery.

Simulation

The code is divided into four parts, three of which are described above and a final compilation and animation. The code uses default integrator in *NDSolve* and breaks each time a change in the system occurs such as impact or mass release. The terminal conditions from each of the simulation serve as the initial conditions for the next. The animation depicts the result which seem exactly like what we would expect intuitively. The code takes less than a minute to run. The helper functions and each of the parts have to be executed independently and sequentially. Part C spits some error sometimes but this goes away when you execute the block again (I was unable to figure out the reason behind this). An extension to this could be to add elastic properties to the string that hold the mass.